

A regra de L'Hôpital

[01] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = 0, \quad \lim_{x \rightarrow p} g_1(x) = 0, \quad \lim_{x \rightarrow p} f_2(x) = 0, \quad \lim_{x \rightarrow p} g_2(x) = 0,$$

mas

$$\lim_{x \rightarrow p} \frac{f_1(x)}{g_1(x)} \neq \lim_{x \rightarrow p} \frac{f_2(x)}{g_2(x)}.$$

[02] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = \infty, \quad \lim_{x \rightarrow p} g_1(x) = \infty, \quad \lim_{x \rightarrow p} f_2(x) = \infty, \quad \lim_{x \rightarrow p} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow p} \frac{f_1(x)}{g_1(x)} \neq \lim_{x \rightarrow p} \frac{f_2(x)}{g_2(x)}.$$

[03] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = 0, \quad \lim_{x \rightarrow p} g_1(x) = \infty, \quad \lim_{x \rightarrow p} f_2(x) = 0, \quad \lim_{x \rightarrow p} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow p} [f_1(x) \cdot g_1(x)] \neq \lim_{x \rightarrow p} [f_2(x) \cdot g_2(x)].$$

[04] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = \infty, \quad \lim_{x \rightarrow p} g_1(x) = \infty, \quad \lim_{x \rightarrow p} f_2(x) = \infty, \quad \lim_{x \rightarrow p} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow p} [f_1(x) - g_1(x)] \neq \lim_{x \rightarrow p} [f_2(x) - g_2(x)].$$

[05] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = 0, \quad \lim_{x \rightarrow p} g_1(x) = 0, \quad \lim_{x \rightarrow p} f_2(x) = 0, \quad \lim_{x \rightarrow p} g_2(x) = 0,$$

mas

$$\lim_{x \rightarrow p} [f_1(x)]^{g_1(x)} \neq \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[06] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = \infty, \quad \lim_{x \rightarrow p} g_1(x) = 0, \quad \lim_{x \rightarrow p} f_2(x) = \infty, \quad \lim_{x \rightarrow p} g_2(x) = 0,$$

mas

$$\lim_{x \rightarrow p} [f_1(x)]^{g_1(x)} \neq \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[07] Encontre funções f_1, g_1, f_2, g_2 , com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x \rightarrow p} f_1(x) = 1, \quad \lim_{x \rightarrow p} g_1(x) = \infty, \quad \lim_{x \rightarrow p} f_2(x) = 1, \quad \lim_{x \rightarrow p} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow p} [f_1(x)]^{g_1(x)} \neq \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[08] Calcule os limites indicados abaixo.

(a) $\lim_{x \rightarrow 1} \frac{x^{64} - 1}{x^{32} - 1},$

(b) $\lim_{x \rightarrow 0} \frac{x + \operatorname{tg}(x)}{\operatorname{sen}(x)},$

(c) $\lim_{x \rightarrow (\pi/2)^+} \frac{\cos(x)}{1 - \operatorname{sen}(x)},$

(d) $\lim_{t \rightarrow 0} \frac{e^t - 1}{t^3},$

(e) $\lim_{t \rightarrow 0} \frac{e^{3t} - 1}{t},$

(f) $\lim_{x \rightarrow 0} \frac{\operatorname{tg}(64x)}{\operatorname{tg}(32x)},$

(g) $\lim_{\theta \rightarrow \pi/2} \frac{1 - \operatorname{sen}(\theta)}{\operatorname{cossec}(\theta)},$

(h) $\lim_{x \rightarrow \infty} \frac{e^x}{x},$

(i) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x},$

(j) $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x},$

(k) $\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))}{x},$

(l) $\lim_{t \rightarrow 0} \frac{5^t - 3^t}{t},$

(m) $\lim_{x \rightarrow 1} \frac{\ln(x)}{\operatorname{sen}(\pi x)},$

(n) $\lim_{x \rightarrow 0} \frac{\operatorname{sen}(x)}{\operatorname{senh}(x)},$

(o) $\lim_{x \rightarrow 0} \frac{\operatorname{arcsen}(x)}{x},$

(p) $\lim_{x \rightarrow 0} \frac{x + \operatorname{sen}(x)}{x + \cos(x)},$

(r) $\lim_{x \rightarrow 0^+} (\sqrt{x} \ln(x)),$

(s) $\lim_{x \rightarrow -\infty} (x^2 e^x),$

(t) $\lim_{x \rightarrow 0} (\cotg(2x) \operatorname{sen}(6x)),$

(u) $\lim_{x \rightarrow 0^-} (\operatorname{sen}(x) \ln(x)),$

(v) $\lim_{x \rightarrow \infty} (x \operatorname{tg}(1/x)),$

(w) $\lim_{x \rightarrow 0} (\operatorname{cossec}(x) - \cotg(x)),$

(y) $\lim_{x \rightarrow \infty} (x^2 - x),$

(z) $\lim_{x \rightarrow \infty} (x - \ln(x)).$

[09] Calcule os limites indicados abaixo.

(a) $\lim_{x \rightarrow 0^+} x^{(x^2)},$

(b) $\lim_{x \rightarrow 0} (1 - 2x)^{1/x},$

(c) $\lim_{x \rightarrow \infty} (1 + 3/x + 5/x^2)^x,$

(d) $\lim_{x \rightarrow \infty} x^{\ln(2)/(1+\ln(x))},$

(e) $\lim_{x \rightarrow \infty} (e^x + x)^{1/x},$

(f) $\lim_{x \rightarrow 0^+} (\cos(x))^{1/x^2}.$

[10] Seja f uma função de classe C^2 . Calcule

$$\lim_{h \rightarrow 0} \frac{f(p+h) - 2f(p) + f(p-h)}{h^2}.$$

Dica: use a regra de L'Hôpital.

Respostas dos Exercícios

[01] Tome $f_1(x) = 4x$, $g_1(x) = 2x$, $f_2(x) = 9x$, $g_2(x) = 3x$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = 0, \quad \lim_{x \rightarrow 0} g_1(x) = 0, \quad \lim_{x \rightarrow 0} f_2(x) = 0, \quad \lim_{x \rightarrow 0} g_2(x) = 0,$$

com

$$\lim_{x \rightarrow 0} \frac{f_1(x)}{g_1(x)} = 2 \neq 3 = \lim_{x \rightarrow 0} \frac{f_2(x)}{g_2(x)}.$$

[02] Tome $f_1(x) = 4/x^2$, $g_1(x) = 2/x^2$, $f_2(x) = 9/x^2$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = \infty, \quad \lim_{x \rightarrow 0} g_1(x) = \infty, \quad \lim_{x \rightarrow 0} f_2(x) = \infty, \quad \lim_{x \rightarrow 0} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow 0} \frac{f_1(x)}{g_1(x)} = 2 \neq 3 = \lim_{x \rightarrow p} \frac{f_2(x)}{g_2(x)}.$$

[03] Tome $f_1(x) = 2x^2$, $g_1(x) = 2/x^2$, $f_2(x) = 3x^2$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = 0, \quad \lim_{x \rightarrow 0} g_1(x) = \infty, \quad \lim_{x \rightarrow 0} f_2(x) = 0, \quad \lim_{x \rightarrow 0} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow 0} [f_1(x) \cdot g_1(x)] = 4 \neq 9 = \lim_{x \rightarrow p} [f_2(x) \cdot g_2(x)].$$

[04] Tome $f_1(x) = 1/x^2$, $g_1(x) = 1/x^2$, $f_2(x) = 1/x^2 + 1$, $g_2(x) = 1/x^2 - 1$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = \infty, \quad \lim_{x \rightarrow 0} g_1(x) = \infty, \quad \lim_{x \rightarrow 0} f_2(x) = \infty, \quad \lim_{x \rightarrow 0} g_2(x) = \infty,$$

mas

$$\lim_{x \rightarrow 0} [f_1(x) - g_1(x)] = 0 \neq 2 = \lim_{x \rightarrow 0} [f_2(x) - g_2(x)].$$

[05] Tome $f_1(x) = e^{-2/x^2}$, $g_1(x) = 2x^2$, $f_2(x) = e^{-3/x^2}$, $g_2(x) = 3x^2$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = 0, \quad \lim_{x \rightarrow 0} g_1(x) = 0, \quad \lim_{x \rightarrow 0} f_2(x) = 0, \quad \lim_{x \rightarrow 0} g_2(x) = 0,$$

com

$$\lim_{x \rightarrow 0} [f_1(x)]^{g_1(x)} = e^{-4} \neq e^{-9} = \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[06] Tome $f_1(x) = e^{2/x^2}$, $g_1(x) = 2x^2$, $f_2(x) = e^{3/x^2}$, $g_2(x) = 3x^2$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = \infty, \quad \lim_{x \rightarrow 0} g_1(x) = 0, \quad \lim_{x \rightarrow 0} f_2(x) = \infty, \quad \lim_{x \rightarrow 0} g_2(x) = 0,$$

com

$$\lim_{x \rightarrow 0} [f_1(x)]^{g_1(x)} = e^4 \neq e^9 = \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[07] Tome $f_1(x) = e^{2x^2}$, $g_1(x) = 2/x^2$, $f_2(x) = e^{3x^2}$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x \rightarrow 0} f_1(x) = 1, \quad \lim_{x \rightarrow 0} g_1(x) = \infty, \quad \lim_{x \rightarrow 0} f_2(x) = 1, \quad \lim_{x \rightarrow 0} g_2(x) = \infty,$$

com

$$\lim_{x \rightarrow 0} [f_1(x)]^{g_1(x)} = e^4 \neq e^9 = \lim_{x \rightarrow p} [f_2(x)]^{g_2(x)}.$$

[08] (a) 2, (b) 2, (c) $-\infty$, (d) $+\infty$, (e) 3, (f) 2, (g) 0, (h) $+\infty$, (i) 0, (j) $-\infty$, (k) 0, (l) $\ln(5/3)$, (m) $-1/\pi$, (n) 1, (o) 1, (p) 0, (r) 0, (s) 0, (t) 3, (u) 0, (v) 1, (w) 0, (y) $+\infty$, (z) $+\infty$.

[09] (a) 1, (b) e^{-2} , (c) e^3 , (d) 2, (e) e , (f) $1/\sqrt{e}$.

[10] $f''(p)$.

Texto composto em \LaTeX 2e, HJB, 27/10/2008.