LISTA DE EXERCÍCIOS
Cálculo I –A–
Humberto José Bortolossi
http://www.professores.uff.br/hjbortol/

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A regra de L'Hôpital

[01] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = 0, \qquad \lim_{x\to p} g_1(x) = 0, \qquad \lim_{x\to p} f_2(x) = 0, \qquad \lim_{x\to p} g_2(x) = 0,$$

mas

$$\lim_{x\to p} \frac{f_1(x)}{g_1(x)} \neq \lim_{x\to p} \frac{f_2(x)}{g_2(x)}.$$

[02] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = \infty, \qquad \lim_{x\to p} g_1(x) = \infty, \qquad \lim_{x\to p} f_2(x) = \infty, \qquad \lim_{x\to p} g_2(x) = \infty,$$

mas

$$\lim_{x\to p} \frac{f_1(x)}{g_1(x)} \neq \lim_{x\to p} \frac{f_2(x)}{g_2(x)}.$$

[03] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = 0, \qquad \lim_{x\to p} g_1(x) = \infty, \qquad \lim_{x\to p} f_2(x) = 0, \qquad \lim_{x\to p} g_2(x) = \infty,$$

mas

$$\lim_{x\to p} [f_1(x) \cdot g_1(x)] \neq \lim_{x\to p} [f_2(x) \cdot g_2(x)].$$

[04] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = \infty, \qquad \lim_{x\to p} g_1(x) = \infty, \qquad \lim_{x\to p} f_2(x) = \infty, \qquad \lim_{x\to p} g_2(x) = \infty,$$

mas

$$\lim_{x\to p} [f_1(x) - g_1(x)] \neq \lim_{x\to p} [f_2(x) - g_2(x)].$$

[05] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = 0, \qquad \lim_{x\to p} g_1(x) = 0, \qquad \lim_{x\to p} f_2(x) = 0, \qquad \lim_{x\to p} g_2(x) = 0,$$

mas

$$\lim_{x\to p} [f_1(x)]^{g_1(x)} \neq \lim_{x\to p} [f_2(x)]^{g_2(x)}.$$

[06] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = \infty, \qquad \lim_{x\to p} g_1(x) = 0, \qquad \lim_{x\to p} f_2(x) = \infty, \qquad \lim_{x\to p} g_2(x) = 0,$$

mas

$$\lim_{x\to p} [f_1(x)]^{g_1(x)} \neq \lim_{x\to p} [f_2(x)]^{g_2(x)}.$$

[07] Encontre funções f_1, g_1, f_2, g_2, com $f_1 \neq f_2$ e $g_1 \neq g_2$, tais que

$$\lim_{x\to p} f_1(x) = 1, \qquad \lim_{x\to p} g_1(x) = \infty, \qquad \lim_{x\to p} f_2(x) = 1, \qquad \lim_{x\to p} g_2(x) = \infty,$$

mas

$$\lim_{x\to p}[f_1(x)]^{g_1(x)} \neq \lim_{x\to p}[f_2(x)]^{g_2(x)}.$$

[08] Calcule os limites indicados abaixo.

(a) $\displaystyle\lim_{x\to 1} \frac{x^{64}-1}{x^{32}-1}$,

(b) $\displaystyle\lim_{x\to 0} \frac{x+\operatorname{tg}(x)}{\operatorname{sen}(x)}$,

(c) $\displaystyle\lim_{x\to(\pi/2)^+} \frac{\cos(x)}{1-\operatorname{sen}(x)}$,

(d) $\displaystyle\lim_{t\to 0} \frac{e^t-1}{t^3}$,

(e) $\displaystyle\lim_{t\to 0} \frac{e^{3\,t}-1}{t}$,

(f) $\displaystyle\lim_{x\to 0} \frac{\operatorname{tg}(64\,x)}{\operatorname{tg}(32\,x)}$,

(g) $\displaystyle\lim_{\theta\to\pi/2} \frac{1-\operatorname{sen}(\theta)}{\operatorname{cossec}(\theta)}$,

(h) $\displaystyle\lim_{x\to\infty} \frac{e^x}{x}$,

(i) $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{x}$,

(j) $\displaystyle\lim_{x\to 0^+} \frac{\ln(x)}{x}$,

(k) $\displaystyle\lim_{x\to\infty} \frac{\ln(\ln(x))}{x}$,

(l) $\displaystyle\lim_{t\to 0} \frac{5^t-3^t}{t}$,

(m) $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{\operatorname{sen}(\pi\,x)}$,

(n) $\displaystyle\lim_{x\to 0} \frac{\operatorname{sen}(x)}{\operatorname{senh}(x)}$,

(o) $\displaystyle\lim_{x\to 0} \frac{\operatorname{arcsen}(x)}{x}$,

(p) $\displaystyle\lim_{x\to 0} \frac{x+\operatorname{sen}(x)}{x+\cos(x)}$,

(r) $\displaystyle\lim_{x\to 0^+} (\sqrt{x}\,\ln(x))$,

(s) $\displaystyle\lim_{x\to-\infty} (x^2\,e^x)$,

(t) $\displaystyle\lim_{x\to 0}(\operatorname{cotg}(2\,x)\,\operatorname{sen}(6\,x))$,

(u) $\displaystyle\lim_{x\to 0^-} (\operatorname{sen}(x)\ln(x))$,

(v) $\displaystyle\lim_{x\to\infty} (x\,\operatorname{tg}(1/x))$,

(w) $\displaystyle\lim_{x\to 0}(\operatorname{cossec}(x)-\operatorname{cotg}(x))$,

(y) $\displaystyle\lim_{x\to\infty} (x^2-x)$,

(z) $\displaystyle\lim_{x\to\infty} (x-\ln(x))$.

[09] Calcule os limites indicados abaixo.

(a) $\displaystyle\lim_{x\to 0^+} x^{(x^2)}$,

(b) $\displaystyle\lim_{x\to 0}(1-2\,x)^{1/x}$,

(c) $\displaystyle\lim_{x\to\infty} (1+3/x+5/x^2)^x$,

(d) $\displaystyle\lim_{x\to\infty} x^{\ln(2)/(1+\ln(x))}$,

(e) $\displaystyle\lim_{x\to\infty} (e^x+x)^{1/x}$,

(f) $\displaystyle\lim_{x\to 0^+} (\cos(x))^{1/x^2}$.

[10] Seja f uma função de classe C^2. Calcule

$$\lim_{h\to 0} \frac{f(p+h)-2\,f(p)+f(p-h)}{h^2}.$$

Dica: use a regra de L'Hôpital.

Respostas dos Exercícios

[01] Tome $f_1(x) = 4\,x$, $g_1(x) = 2\,x$, $f_2(x) = 9\,x$, $g_2(x) = 3\,x$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = 0, \qquad \lim_{x\to 0} g_1(x) = 0, \qquad \lim_{x\to 0} f_2(x) = 0, \qquad \lim_{x\to 0} g_2(x) = 0,$$

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$$\lim_{x\to 0} \frac{f_1(x)}{g_1(x)} = 2 \neq 3 = \lim_{x\to 0} \frac{f_2(x)}{g_2(x)}.$$

[02] Tome $f_1(x) = 4/x^2$, $g_1(x) = 2/x^2$, $f_2(x) = 9/x^2$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = \infty, \qquad \lim_{x\to 0} g_1(x) = \infty, \qquad \lim_{x\to 0} f_2(x) = \infty, \qquad \lim_{x\to 0} g_2(x) = \infty,$$

mas

$$\lim_{x\to 0} \frac{f_1(x)}{g_1(x)} = 2 \neq 3 = \lim_{x\to p} \frac{f_2(x)}{g_2(x)}.$$

[03] Tome $f_1(x) = 2\,x^2$, $g_1(x) = 2/x^2$, $f_2(x) = 3\,x^2$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = 0, \qquad \lim_{x\to 0} g_1(x) = \infty, \qquad \lim_{x\to 0} f_2(x) = 0, \qquad \lim_{x\to 0} g_2(x) = \infty,$$

mas

$$\lim_{x\to 0} [f_1(x)\cdot g_1(x)] = 4 \neq 9 = \lim_{x\to p} [f_2(x)\cdot g_2(x)].$$

[04] Tome $f_1(x) = 1/x^2$, $g_1(x) = 1/x^2$, $f_2(x) = 1/x^2 + 1$, $g_2(x) = 1/x^2 - 1$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = \infty, \qquad \lim_{x\to 0} g_1(x) = \infty, \qquad \lim_{x\to 0} f_2(x) = \infty, \qquad \lim_{x\to 0} g_2(x) = \infty,$$

mas

$$\lim_{x\to 0} [f_1(x) - g_1(x)] = 0 \neq 2 = \lim_{x\to 0} [f_2(x) - g_2(x)].$$

[05] Tome $f_1(x) = e^{-2/x^2}$, $g_1(x) = 2\,x^2$, $f_2(x) = e^{-3/x^2}$, $g_2(x) = 3\,x^2$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = 0, \qquad \lim_{x\to 0} g_1(x) = 0, \qquad \lim_{x\to 0} f_2(x) = 0, \qquad \lim_{x\to 0} g_2(x) = 0,$$

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$$\lim_{x\to 0} [f_1(x)]^{g_1(x)} = e^{-4} \neq e^{-9} = \lim_{x\to p} [f_2(x)]^{g_2(x)}.$$

[06] Tome $f_1(x) = e^{2/x^2}$, $g_1(x) = 2\,x^2$, $f_2(x) = e^{3/x^2}$, $g_2(x) = 3\,x^2$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = \infty, \qquad \lim_{x\to 0} g_1(x) = 0, \qquad \lim_{x\to 0} f_2(x) = \infty, \qquad \lim_{x\to 0} g_2(x) = 0,$$

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$$\lim_{x\to 0} [f_1(x)]^{g_1(x)} = e^{4} \neq e^{9} = \lim_{x\to p} [f_2(x)]^{g_2(x)}.$$

[07] Tome $f_1(x) = e^{2\,x^2}$, $g_1(x) = 2/x^2$, $f_2(x) = e^{3\,x^2}$, $g_2(x) = 3/x^2$ e $p = 0$. Note que

$$\lim_{x\to 0} f_1(x) = 1, \qquad \lim_{x\to 0} g_1(x) = \infty, \qquad \lim_{x\to 0} f_2(x) = 1, \qquad \lim_{x\to 0} g_2(x) = \infty,$$

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$$\lim_{x\to 0} [f_1(x)]^{g_1(x)} = e^{4} \neq e^{9} = \lim_{x\to p} [f_2(x)]^{g_2(x)}.$$

[08] (a) 2, (b) 2, (c) $-\infty$, (d) $+\infty$, (e) 3, (f) 2, (g) 0, (h) $+\infty$, (i) 0, (j) $-\infty$, (k) 0, (l) $\ln(5/3)$, (m) $-1/\pi$, (n) 1, (o) 1, (p) 0, (r) 0, (s) 0, (t) 3, (u) 0, (v) 1, (w) 0, (y) $+\infty$, (z) $+\infty$.

[09] (a) 1, (b) e^{-2}, (c) e^3, (d) 2, (e) e, (f) $1/\sqrt{e}$.

[10] $f''(p)$.

Texto composto em LaTeX2e, HJB, 04/06/2009.